

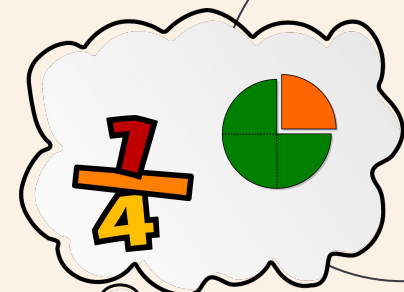
# Using Visualization to Unlock Fraction Sense



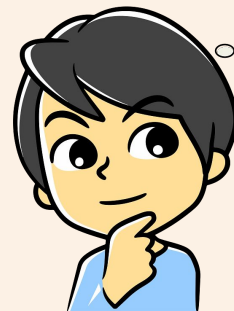
NCTM 2023

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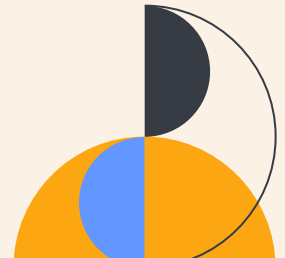
<https://bit.ly/nctmfractionsense>



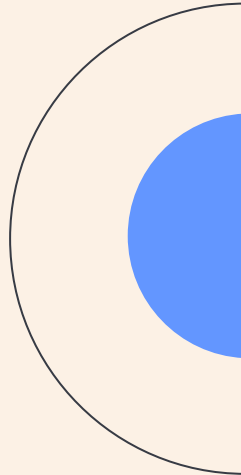


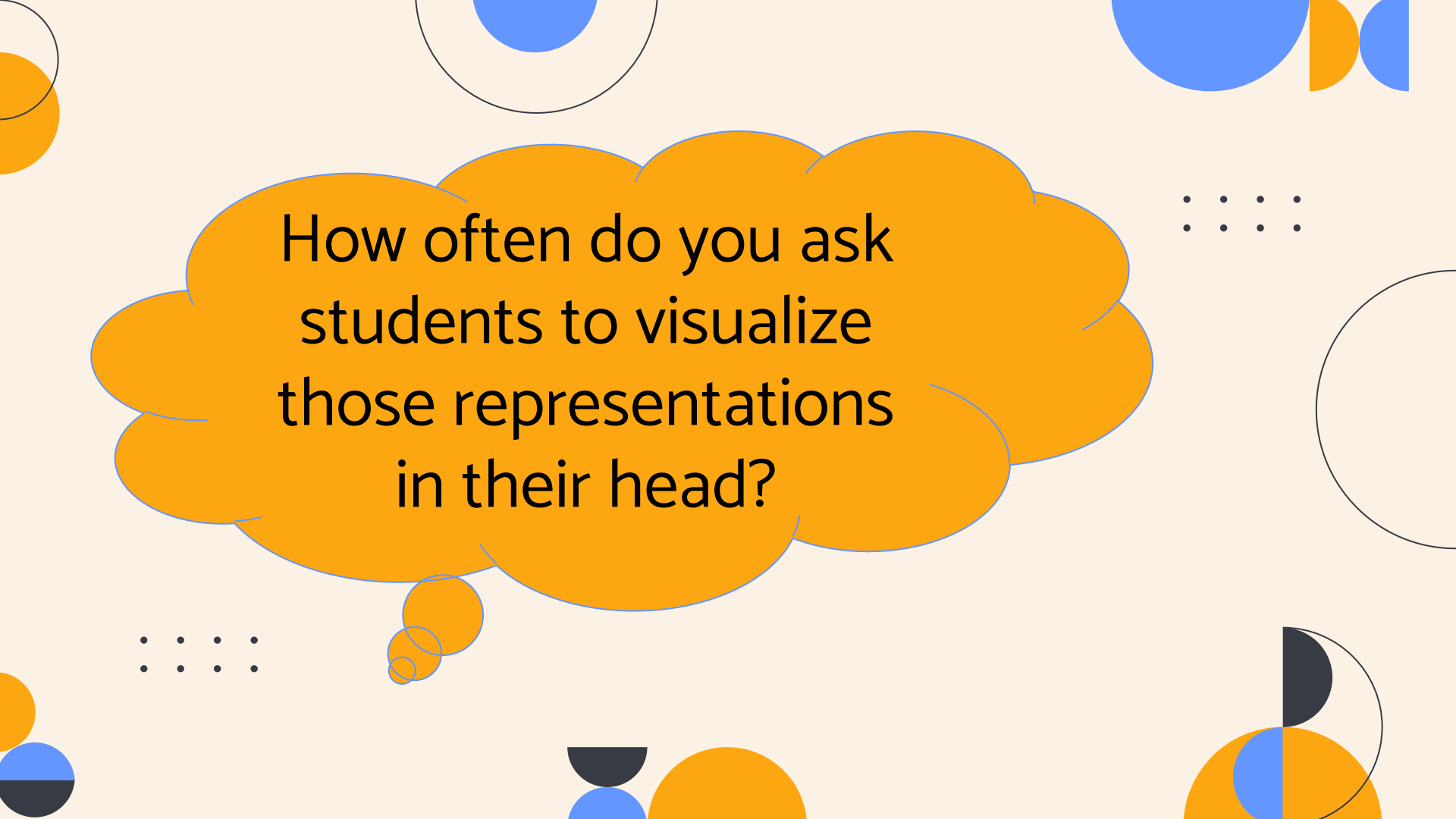
# Agenda:

- 1. Visual Representations**
- 2. What is Fraction Sense**
- 3. Case Study in Visualization**
- 4. Manipulatives**
- 5. Number Talks**
- 6. Games**

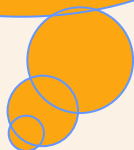
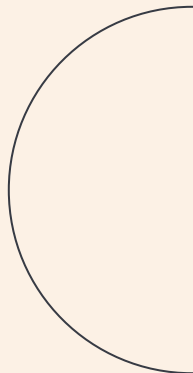


**What visual representations do  
you use in your math class today?**






How often do you ask  
students to visualize  
those representations  
in their head?



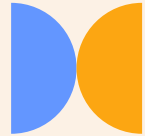
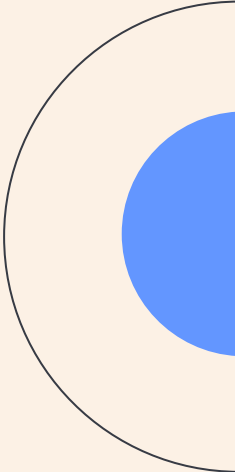




# Two Different Types of Visual Representations

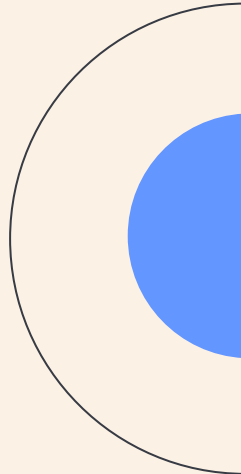


When you are talking about visual representation in mathematics, you may be talking about **representing information on a page** with a diagram or chart, or **representing information in your head with an image**.



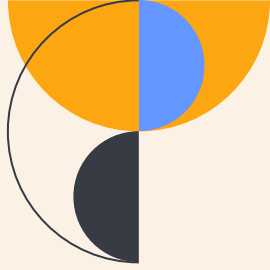
# Visual Representations are...

- retrievable
- flexible
- used across grade levels
- used across types of problems
- an accurate depiction of a given problem's mathematical quantities and relationships
- a student's understanding of the problem
- used to improve mathematical reasoning skills





# Research shows

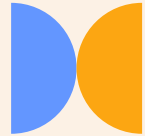
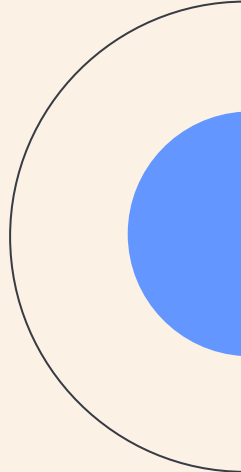
- 
- Students who use accurate visual representations are **six times more likely to correctly solve mathematics problems** than students who do not use them.  
*(Boonen, van Wesel, Jolles, & van der Schoot, 2014)*
  - Students with a learning disability (LD) often do not create accurate visual representations or use them strategically to solve problems. **Teaching students to systematically use a visual representation to solve word problems has led to substantial improvements in math achievement** for students with learning disabilities.  
*(van Garderen, Scheuermann, & Jackson, 2012; van Garderen, Scheuermann, & Poch, 2014)*



# Why Visualize Fractions?

The concept of breaking a number into parts can be too abstract for some students to grasp.

Creating visual representations of fractions helps them become more tangible, and enables students to see the connections between parts and whole more clearly.



# What is Fraction Sense?



Fraction sense implies a deep and flexible understanding of fractions that is not dependent on any one context or type of problem. Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don't apply rules and procedures blindly; nor do they give nonsensical answers to problems involving fractions.


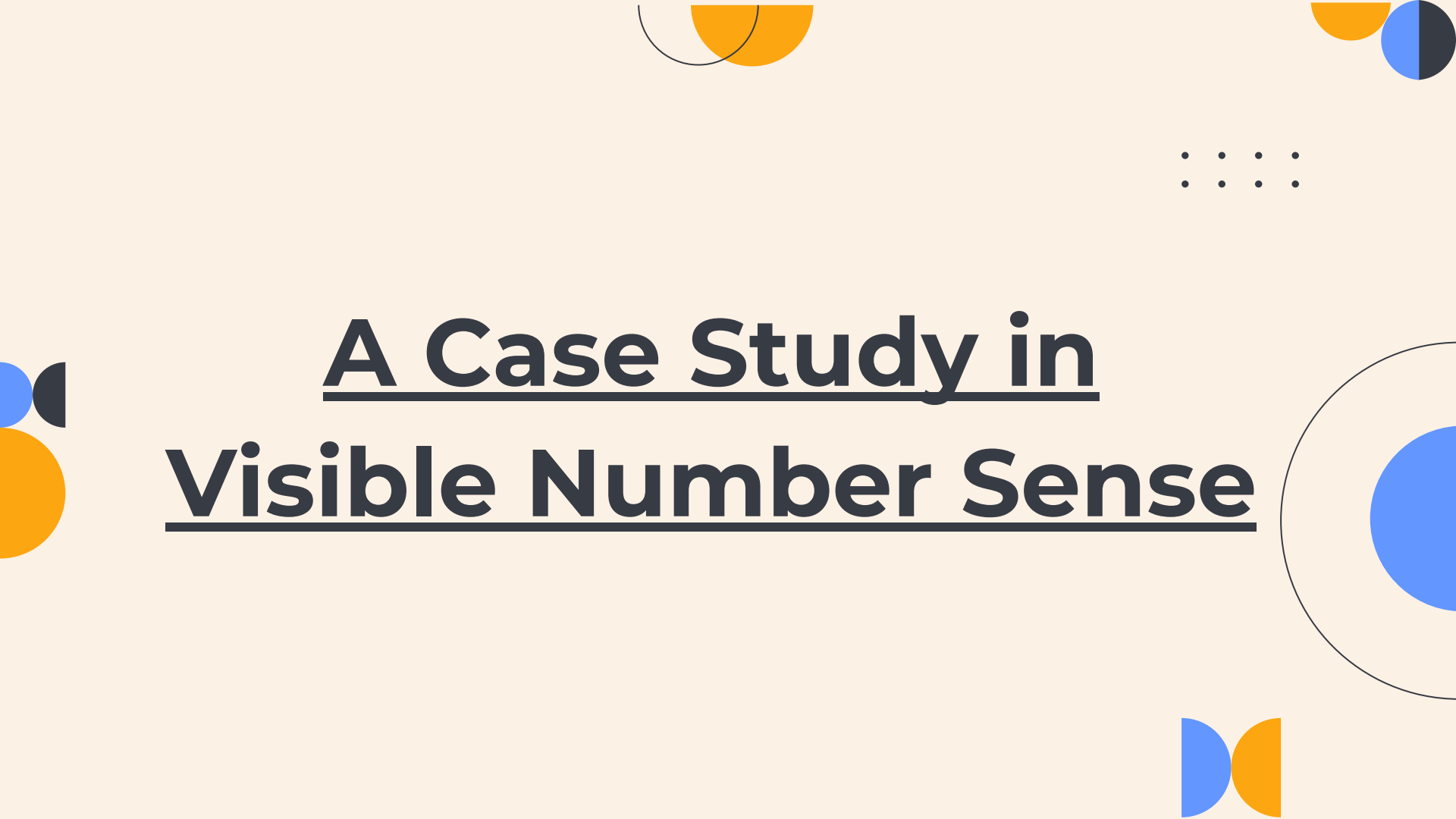
- Julie McNamara & Meghan M. Shaughnessy  
*Beyond Pizzas and Pies*

# Why is Fraction Sense Important?

## Memorization leads to:

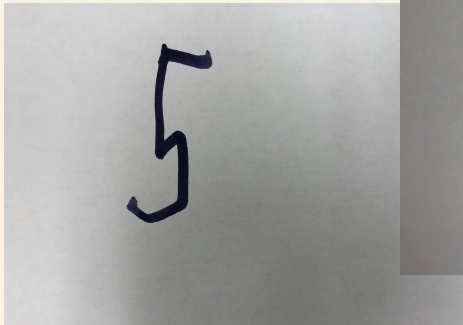
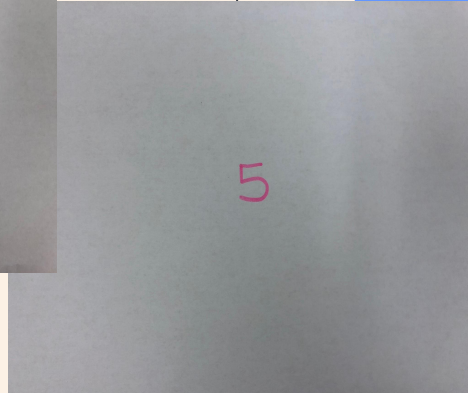
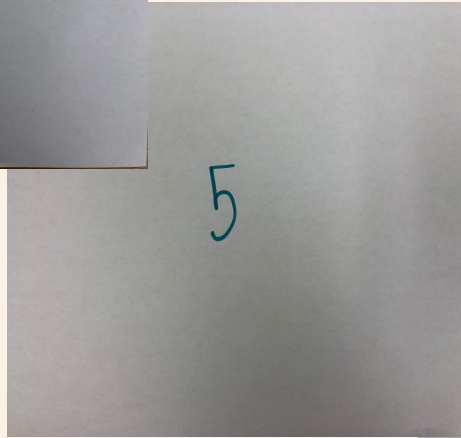
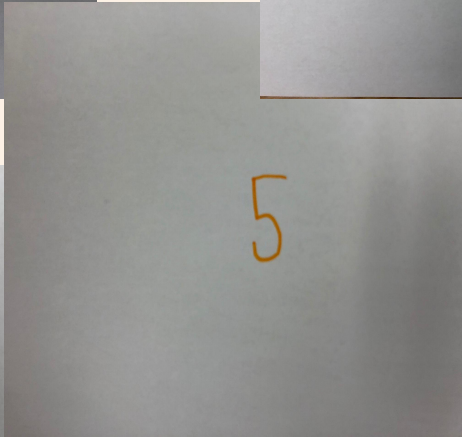
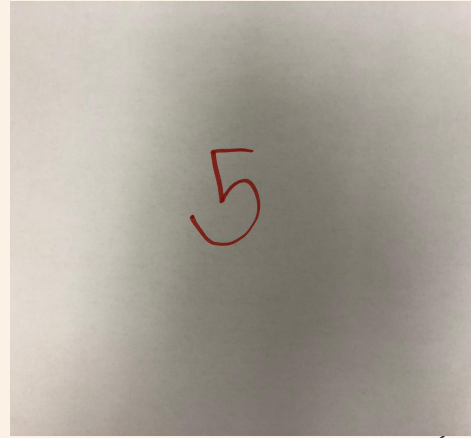
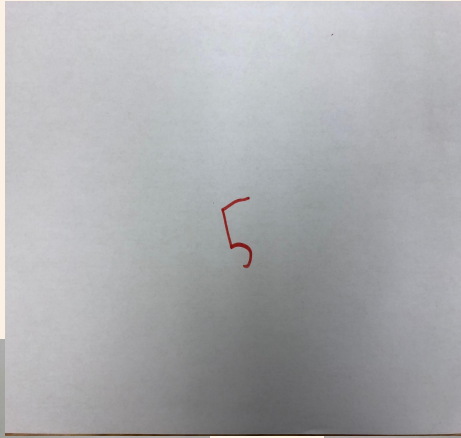
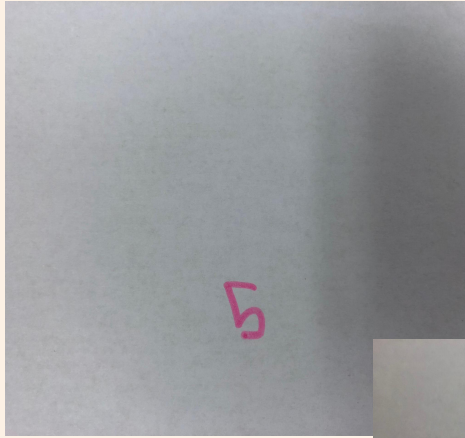
- lack of reasoning
- lack of thought
- lack of number sense





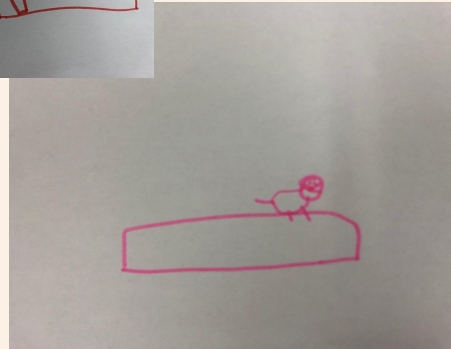
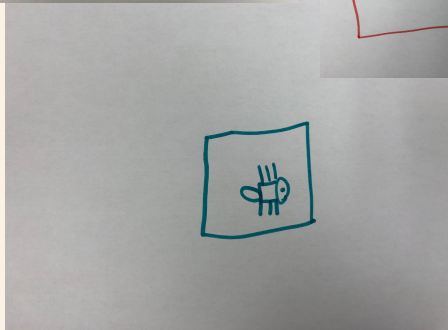
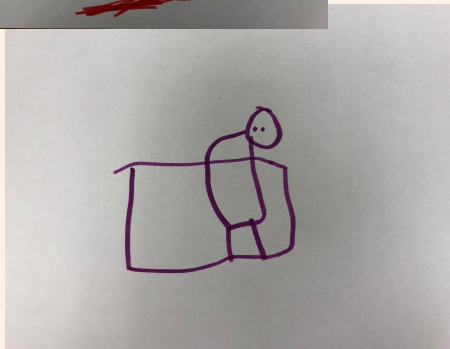
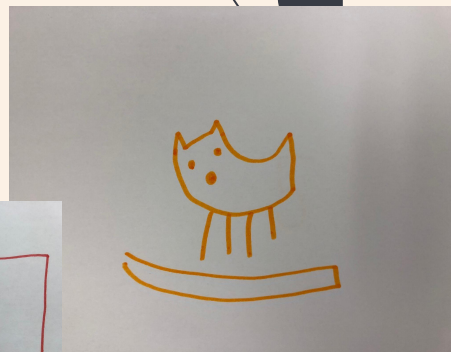
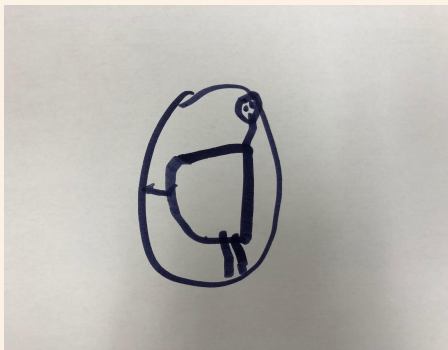
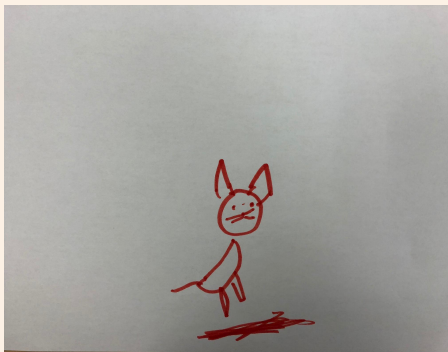
**A Case Study in**  
**Visible Number Sense**

# How Do You See “5”?

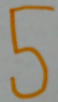




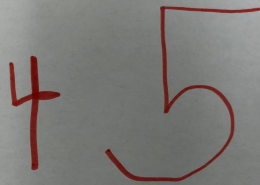
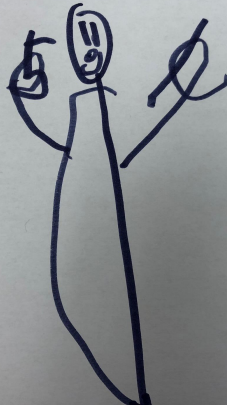
# How Do You See “The Cat is on the Mat”?



# Make a Picture of 5 in Your Mind...



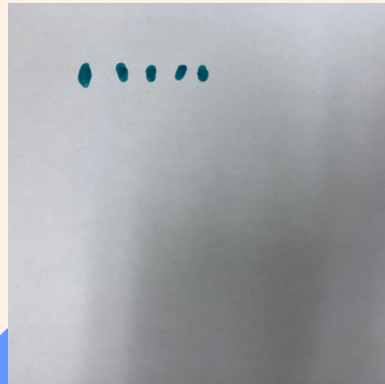
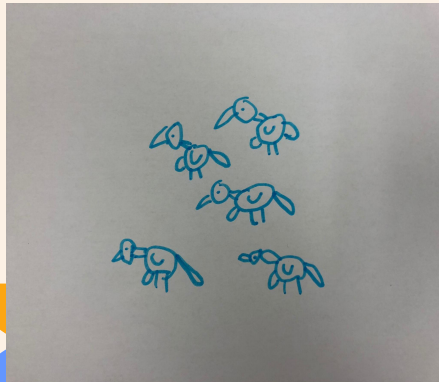
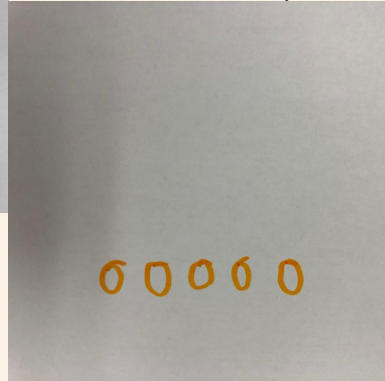
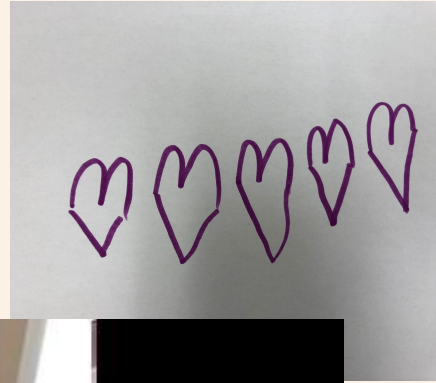
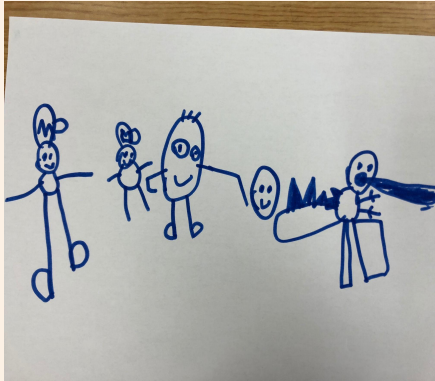
5



4 5



# What We Want Students to See...



# Let's Try it with Fractions....

$$\frac{3}{4}$$

34

three fourths

$$\frac{3}{4}$$

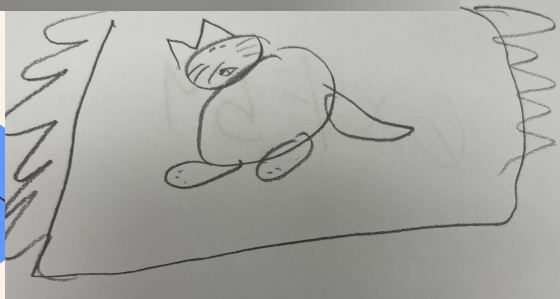
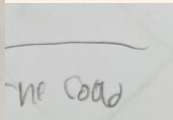
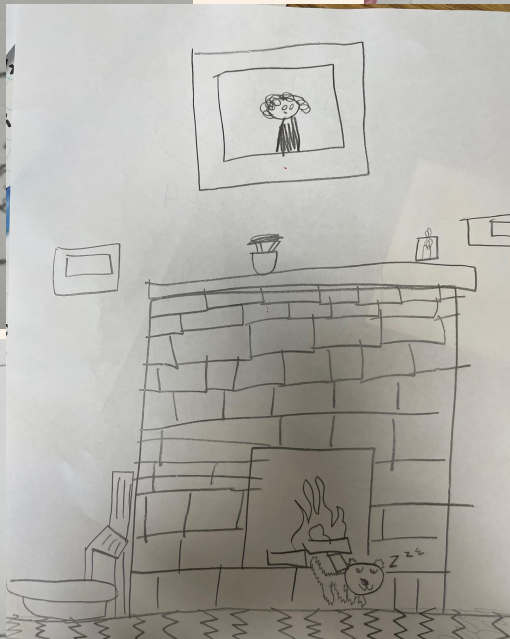
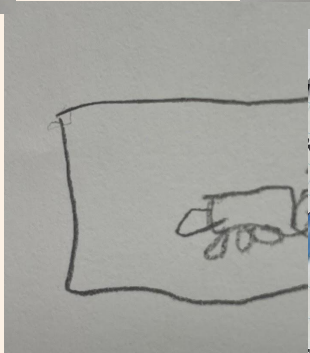
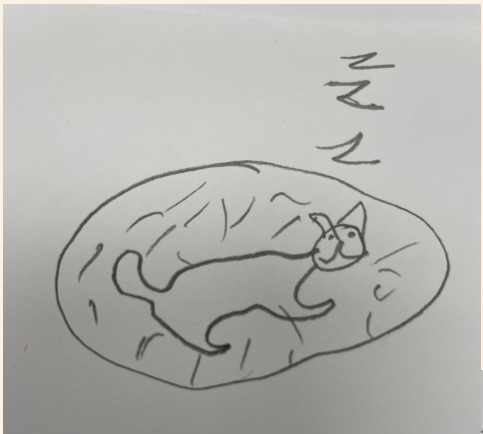
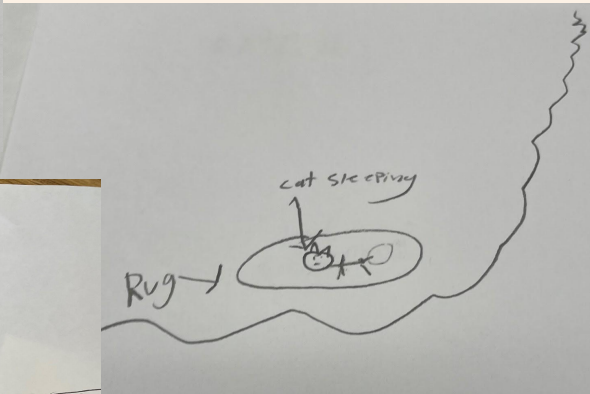
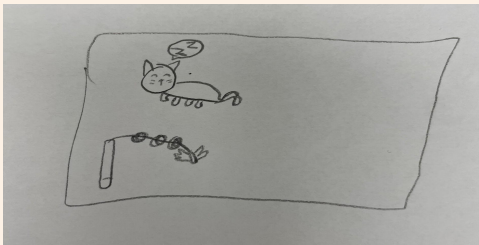
3  
4

$$\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4}$$

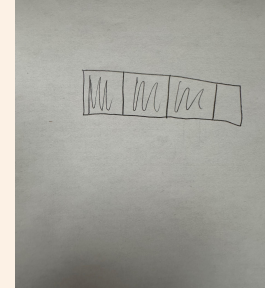
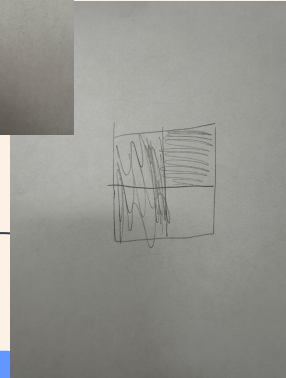
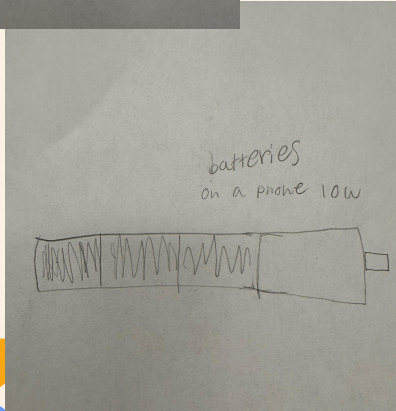
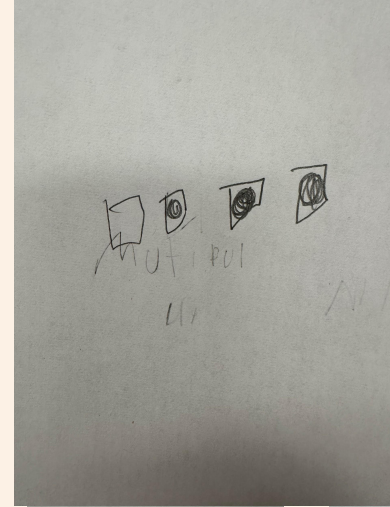
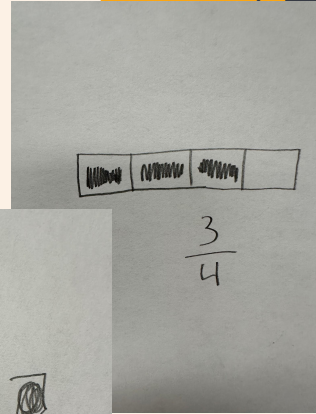
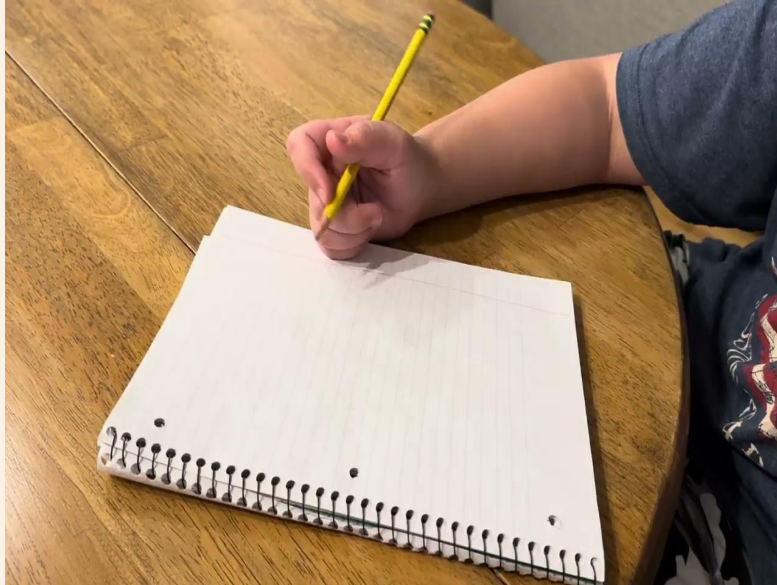
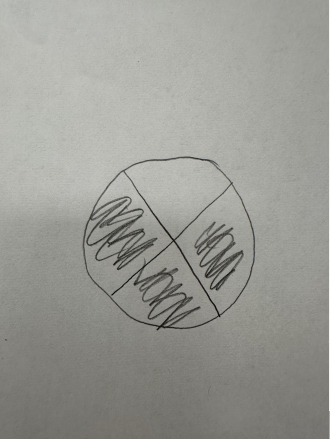
4 4 4

⋮ ⋮ ⋮ ⋮





# What We Want Students to See?



# How Can We Change This?

- Engage students in activities where they are actively creating mental images
- Use of manipulatives
- Reasoning activities
- Multiple representations
- Discourse





# Manipulatives

Provide a concrete and visual example of the fractional parts and whole.



# Notice and Wonder



Fraction Tiles Investigation

I NOTICE	I WONDER



# Cuisenaire Rods



digital cuisenaire rods

## Part to Whole and Whole to Part

Date: \_\_\_\_\_

Name: \_\_\_\_\_

1. Start with the orange rod.

a. Which rod is  $\frac{1}{2}$  of the orange rod? \_\_\_\_\_  
How do you know?

b. Which rod is  $\frac{1}{3}$  of the orange rod? \_\_\_\_\_  
How do you know?

c. Which rod is  $\frac{1}{10}$  of the orange rod? \_\_\_\_\_  
How do you know?

2. Take out the brown rod.

a. Which rod is  $\frac{1}{2}$  of the brown rod? \_\_\_\_\_  
How do you know?

b. Which rod is  $\frac{1}{4}$  of the brown rod? \_\_\_\_\_  
How do you know?

c. Which rod is  $\frac{1}{8}$  of the brown rod? \_\_\_\_\_  
How do you know?

3. Take out the light-green rod.

a. If the light-green rod is  $\frac{1}{3}$ , which rod is the whole?  
\_\_\_\_\_

b. If the light-green rod is  $\frac{1}{3}$ , which rod is  $\frac{2}{3}$ ?  
\_\_\_\_\_

4. Take out the white rod.

a. If the white rod is  $\frac{1}{5}$ , which rod is the whole?  
\_\_\_\_\_

b. If the white rod is  $\frac{1}{5}$ , which rod is  $\frac{2}{5}$ ? \_\_\_\_\_  
How do you know?

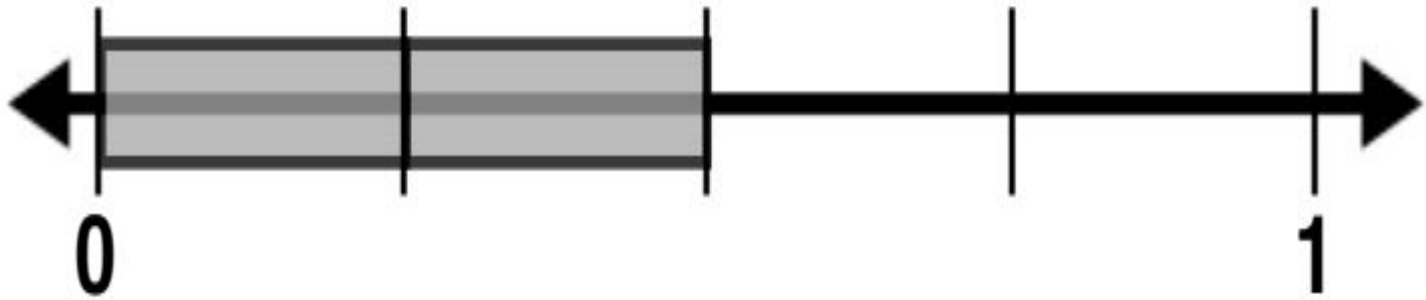


# Number Talks

Help to build conceptual understanding,  
flexibility, and the ability to reason.



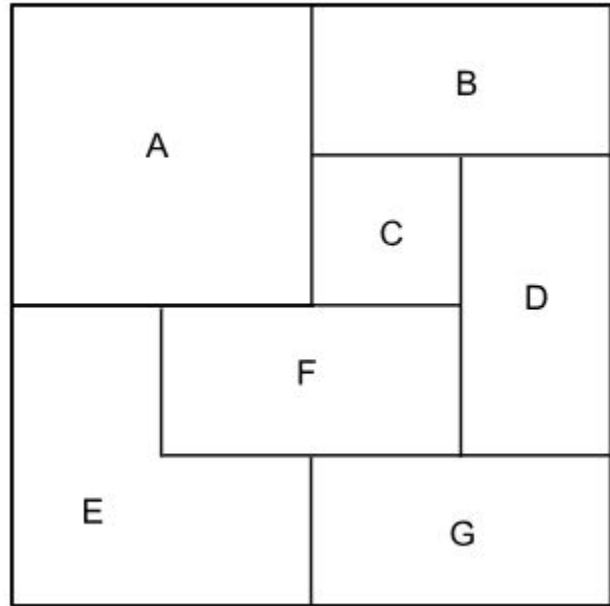
# Number Talks - Grade 3 Example



# Number Talks

Pablo's birthday cake was cut into 7 pieces.  
What fraction of the cake is each piece?

If students ask, you can tell them that the vertical line between A and B is half way between the two sides of the square, etc.

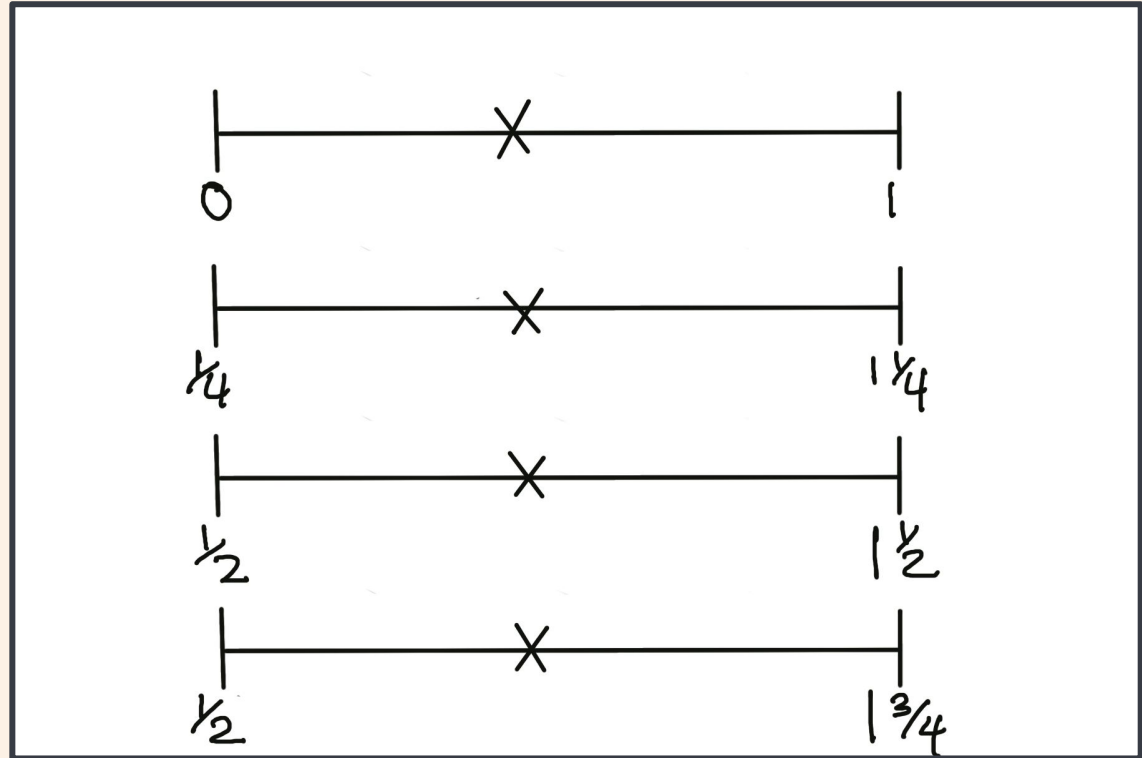




# Number Talks - Grade 5

Number Strings:

What is the X?



source: G. Fletcher

# Fraction Cards (Deluxe)

halves, thirds, quarters, sixths, & eighths



\*\*\*12 Bonus Misconception Cards Included\*\*\*

Created by Graham Fletcher  
@gfletchy  
[www.gfletchy.com](http://www.gfletchy.com)





# Games

Games encourage mathematical thinking that leads to finding different strategies for solving problems and deepening a student's understanding of numbers.

# What's My Fraction?

You will need: colored square tiles, folder, fraction bar recording sheet, and crayons

1. Pick a partner. Choose 6 color tiles. Be sure to have at least 2 different colors of tiles.
2. Put a folder between you so you cannot see each other's work and build a fraction bar with the tiles.
3. Use fractions to describe your fraction bar to your partner. Based on your fraction clues, your partner should build a replica of your fraction bar.

Example:

$\frac{1}{3}$  of my fraction bar is green.



4. Record your work.
5. Now it's your partner's turn to build a fraction bar and provide you clues to build a replica of it!
6. Repeat, this time building a fraction bar using 8 tiles, 10 tiles, and 12 tiles..

## Try it at your table

### What's My Fraction? Recording Sheet

3.G.2

--	--	--	--	--	--	--

# of Tiles	Fraction Clue #1	Fraction Clue #2	Fraction Clue #3
6			

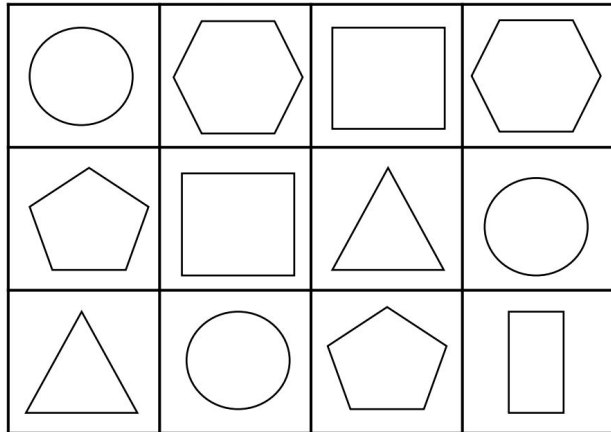
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# of Tiles	Fraction Clue #1	Fraction Clue #2	Fraction Clue #3
8			

## What's My Fraction

# Try it at your table

Guess My Fractions Game Board



## Guess My Fractions

You will need: game board, folder to put between you and your partner, crayons or colored pencils

You will be working with a partner to guess each other's fractions.

1. Put a folder between your game board and your partner's game board. Do not let your partner see what you shade!
2. Player A partitions each shape into equal parts and shades in a fraction of each shape. Label each shape with the fraction of shaded parts.
3. Player A will give instructions to his or her partner on how to shade in his/her board to look exactly like Player A's board.
4. Remove the folder that was between your boards and check to see if they match. Discuss why or why not you were able to accurately complete the task.
5. Players will now switch roles and repeat steps 1-4.

# Pattern Block Fractions

Try it at  
your table









**The Whole Matters!**

**If we change the  
whole... what is the  
value of each pattern  
block.**

**Pattern Block Fractions**

Date: \_\_\_\_\_

Name: \_\_\_\_\_

	 Whole	 Whole	 Whole	 Whole
				
				
				
				

# Fraction Blackout

You will need: one game board for each player, fraction cards


1. Play with a partner.
2. Place the fraction cards facedown in a pile between you and your partner.
3. On your turn, choose a fraction card from the pile.
  - If you have that fraction on your game board, shade it in, once. You can also shade equivalent fractions. For example, if you choose  $\frac{1}{2}$ , you can shade in four  $\frac{1}{8}$ . You must tell your partner why that works.
  - If you cannot shade in that fraction or an equivalent fraction, your turn is over.
4. Continue taking turns until there are no more cards in the pile. The player with the greatest number of one wholes wins the game.

## Fraction Blackout



$\frac{1}{2}$	$\frac{2}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{1}{5}$
$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{1}{6}$

## Decompose a Fraction


1. Roll 2 dice to make a fraction. The fraction can be greater than one.
2. Decompose the fraction into a sum of fractions with the same denominator. (at least 3 different decompositions)
3. Record each decomposition with an equation and a shaded area model.

Example:  $\frac{4}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$  

The area model shows a rectangle divided into 6 equal vertical strips. The first four strips are shaded in different colors: yellow, blue, orange, and green. The last two strips are white.

$\frac{4}{6} = \frac{1}{6} + \frac{3}{6}$  

The area model shows a rectangle divided into 6 equal vertical strips. The first strip is shaded yellow, and the next three strips are shaded blue. The last two strips are white.

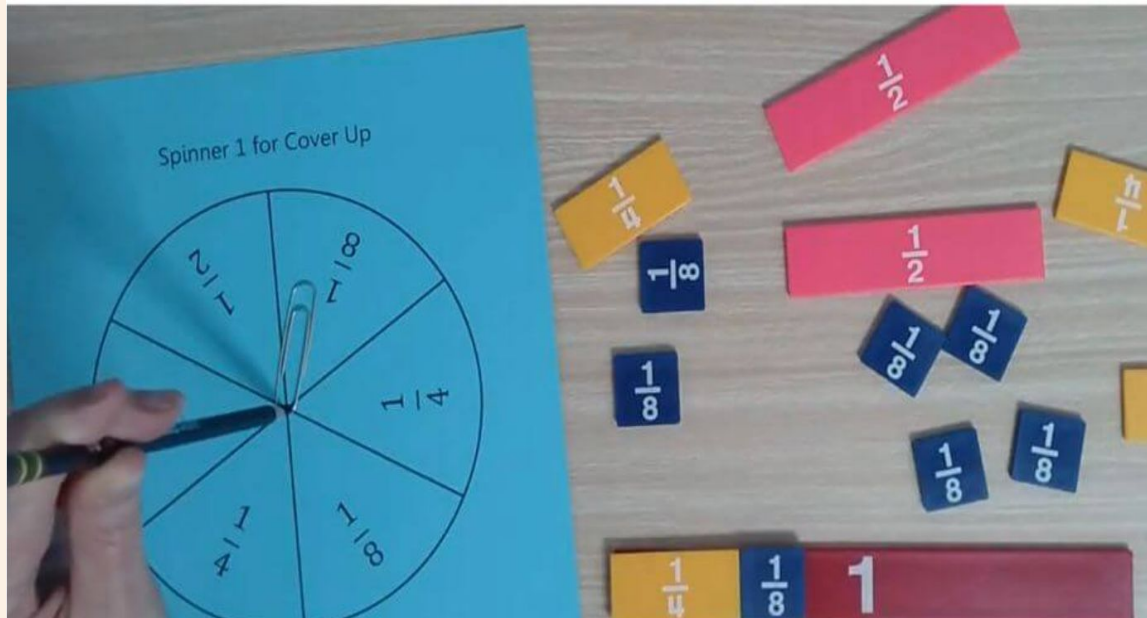
$\frac{4}{6} = \frac{2}{6} + \frac{2}{6}$  

The area model shows a rectangle divided into 6 equal vertical strips. The first two strips are shaded yellow, and the next two strips are shaded blue. The last two strips are white.

Decompose a Fraction

# Cover Up

Be the first player to cover the whole



Add Up/Take Down... adding and subtracting fractions

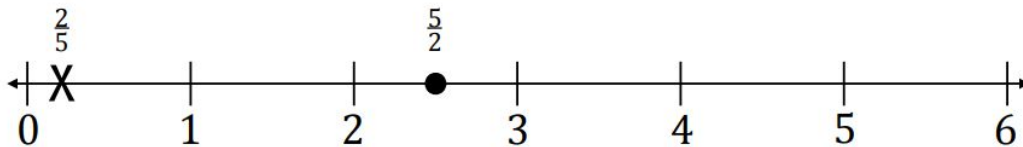
cover up spinner

# Dotty Pairs

Try it at  
your table

## Dotty Pairs Game

- Students play in pairs. One student uses dots as their indicator on the board and the other player takes crosses. Different colored pens can also be used. *Note: Teams of 2 players will also encourage more discourse and conversation amongst team members.*
- Players take turns rolling 2 dice. The numbers are used to form a fraction which can be less than OR greater than one. (Example: If 2 and 5 are rolled, the fractions  $\frac{2}{5}$  or  $\frac{5}{2}$  can be made.)
- One of the two fractions is chosen and marked on a 0–6 number line with the player's identifying mark (dot or cross).
- Players take turns. The aim of the game is for one player to get three marks uninterrupted by their opponent's marks on the number line.





# Greg Tang - NumTanga (level 5)

**TANG MATH**

EXIT TANGA!

LEVEL 5  
ROUND 1

SCORE: 3

YOU

TIME

three fourths

$\frac{3}{4}$

The game interface features two rocket-shaped columns. The left column contains the text 'three fourths' and a circle divided into four equal sectors with one sector shaded red. The right column contains a ten-frame with three orange blocks in the top row and four empty white blocks in the bottom row, and the fraction  $\frac{3}{4}$  below it. On the left side of the screen, there is a vertical column of ten circles: the top seven are white and the bottom three are black. On the right side, there is a vertical column of ten circles: the top seven are white and the bottom three are black. The background is orange with stylized rocket flames.

NumTanga

# Thanks!



Do you have any questions?

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rtedesco@trumbullps.org

<https://bit.ly/nctmfractionsense>

